

ity would be to use  $\text{Co}^{2+}$  substitutions in octahedral sites (usually with  $\text{Si}^{4+}$  in tetrahedral sites for charge compensation) to alter the cubic anisotropy constant (12) and provide either  $K_1 = 0$  for  $\{111\}$  plane applications or  $K_1 > 0$  for the  $\{001\}$  planes.

For spinels, the  $\lambda_{100}$  constant is normally large and negative ( $\sim -20 \times 10^{-6}$ ), while  $\lambda_{111}$  is small and positive ( $\sim +0.5 \times 10^{-6}$ ). Trivalent manganese may be used to change the signs of both constants (13,14) and was effective in producing uniaxial anisotropy by compressive stress for a  $\text{Mn}_{0.7}\text{Fe}_{2.3}\text{O}_4$  epitaxial film deposited on an MgO substrate with a (110) orientation (4). Divalent cobalt is known to be extremely effective in altering  $K_1$  in spinels (13) and could be a useful additive in designing compositions with zero or positive  $K_1$  values for possible compressive stress applications.

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### Appendix

For three mutually orthogonal sets of direction cosines,  $\gamma_i$ ,  $\gamma_i'$ , and  $\beta_i$ ,

$$\sum_i \gamma_i \gamma_i' = \sum_i \gamma_i \beta_i = \sum_i \gamma_i' \beta_i = 0. \quad (A1)$$

From basic analytical geometry theory, these cosines are related by

$$\begin{aligned} \gamma_1 &= \beta_2 \gamma_3' - \beta_3 \gamma_2' & \gamma_1' &= \beta_2 \gamma_3 - \beta_3 \gamma_2 \\ \gamma_2 &= \beta_3 \gamma_1' - \beta_1 \gamma_3' & \gamma_2' &= \beta_3 \gamma_1 - \beta_1 \gamma_3 \\ \gamma_3 &= \beta_1 \gamma_2' - \beta_2 \gamma_1' & \gamma_3' &= \beta_1 \gamma_2 - \beta_2 \gamma_1 \end{aligned} \quad (A2)$$

By appropriate substitution from among the relations of Eq. (A2), it may be readily shown that

$$\gamma_1^2 = \beta_2^2 (\beta_1 \gamma_2 - \beta_2 \gamma_1)^2 - \beta_3^2 (\beta_3 \gamma_1 - \beta_1 \gamma_3)^2 \quad (A3)$$

and

$$\gamma_1'^2 = (\beta_2 \gamma_3 - \beta_3 \gamma_2)^2. \quad (A4)$$

By adding Eqs. (A3) and (A4) and simplifying by means of Eq. (A1) in addition to the normality conditions of the cosines, it may be easily shown that the generalized result is given by

$$\gamma_1^2 + \gamma_1'^2 = 1 - \beta_1^2. \quad (A5)$$

By similar applications of the relations in Eq. (A2),

$$\gamma_1 \gamma_2 = \gamma_2^2 \beta_1 \beta_2 + \gamma_2 \gamma_3 \beta_1 \beta_3 - \gamma_1 \gamma_2 (\beta_2^2 + \beta_3^2) \quad (A6)$$

and

$$\gamma_1 \gamma_2' = \gamma_1 \gamma_3 \beta_2 \beta_3 - \gamma_3^2 \beta_1 \beta_2 - \gamma_1 \gamma_2 \beta_3^2 + \gamma_2 \gamma_3 \beta_1 \beta_3. \quad (A7)$$

After reductions and simplifications of the type employed in the derivation of Eq. (A5), the sum of Eqs. (A6) and (A7) becomes, after generalization,

$$\gamma_i \gamma_j + \gamma_i' \gamma_j' = -\beta_i \beta_j. \quad (A8)$$